A Survey on Families of Binary Sequences

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In this correspondence, we mention several families of binary m-sequences which are already introduced in many articles. Most of the families have three valued non-trivial auto and cross correlations But in few cases they have five and six valued nontrivial correlations.

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I. INTRODUCTION

It has been well established that families of binary sequences with low correlation have important applications in code-division multiple access (CDMA), communication systems and cryptographic system([1],[2],[3]). To check the optimality of the sequence families, we have Sidelnikov's bound([9]). It states that for any family of k binary sequences of period N_k, if $k \geq N_k$, then

$$R_{max} \geq (2N_k - 2)^{\frac{1}{2}},$$

where R_{max} is the maximum magnitude of correlation values except for the in-phase autocorrelation value. The well-known Gold's family ([5]) is a binary sequence family which satisfies Sidelnikov's bound. It has correlations $2^n - 1, -1, -1 \pm 2^{\frac{n+1}{2}}$, where n is odd. But Gold sequence cannot resist attacks based on Berlekamp-Massey algorithm due to its small linear span. So the Gold-like families with larger linear span were constructed.

Boztas and Kumar[4] discovered the odd case of Gold-like sequence family. The correlations of their families are identical to those of Gold sequences, namely $\{2^n - 1, -1, -1 \pm 2^{\frac{n+1}{2}}\}$.

For even n, Udaya[13] introduced families of binary sequences with correlations $2^n - 1, -1, -1 \pm 2^{\frac{n}{2}}, -1 \pm 2^{\frac{n}{2}+1}$ which corresponds to even case of Gold-like sequence family.

The generalization of Gold-like sequences were done by Kim and No [7]. They have introduced GKW-like sequences by using the quadratic form technique and constructed families with correlations $2^n - 1, -1, -1 \pm 2^{\frac{n+e}{2}}$ and $2^n - 1, -1, -1 \pm 2^{\frac{n}{2}}, -1 \pm 2^{\frac{n}{2}+e}$ respectively, where n and e are positive integers, $e|n$.

Later Wang and Qi [14] introdudced two new families S_1 and S_2 which are optimal by Sidelnikov bound.

In their work [10–12], they have combined the trace forms mentioned in [4, 7] and introduced new families. Some of them are actually Gold-like family.

II. PRELIMINARIES

Let $\mathbb{F}_{2^n}$ be the finite field with 2^n elements. Then the trace function from $\mathbb{F}_{2^n}$ to $\mathbb{F}_{2^m}$ is defined by

$$tr_m^n(x) = \sum_{i=0}^{\frac{n}{m}-1} x^{2^{mi}}$$

where $x \in \mathbb{F}_{2^n}$ and $m|n$. The trace function has the following properties:

1. $tr_m^n(ax + by) = atr_m^n(x) + btr_m^n(x)$, for all $a, b \in \mathbb{F}_{2^m}, x, y \in \mathbb{F}_{2^n}$;
2. $tr_m^n(x^{2^m}) = tr_m^n(x)$, for all $x \in \mathbb{F}_{2^n}$.

Let $f(x)$ be a function from $\mathbb{F}_{2^n}$ to $\mathbb{F}_2$ and $\lambda \in F_{2^n}$. The *trace transform* $F(\lambda)$ of $f(x)$ is defined by

$$F(\lambda) = \sum_{x \in \mathbb{F}_{2^n}} (-1)^{f(x) + tr_1^n(x\lambda)}.$$

Definition 1. *Let $x = \sum_{i=1}^{n} x_i \alpha_i$, where $x_i \in \mathbb{F}_2$ and $\alpha_i, i = 1, 2, ..., n$, is a basis for $\mathbb{F}_{2^n}$ over $\mathbb{F}_2$. Then the function $f(x)$ over $\mathbb{F}_{2^n}$ to $\mathbb{F}_2$ is a quadratic form if it can be expressed as*

$$f(x) = f(\sum_{i=1}^{n} x_i \alpha_i) = \sum_{i=1}^{n} \sum_{j=1}^{n} b_{i,j} x_i x_j,$$

where $b_{i,j} \in \mathbb{F}_{2^n}$.

The quadratic form has been well analized in [8]. We also recall that the symplectic bilinear form of a quadratic form $f(x)$ is

$$B(x, z) = f(x) + f(z) + f(x + z) \text{ for } x, z \in \mathbb{F}_{2^n}.$$

Finding the dimension of the radical of a quadratic form is very crucial to find the correlations of binary sequences.

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The radical of the quadratic form $f(x)$ is the number of solutions of $x \in \mathbb{F}_{2^n}$ to

$$B(x,z) = f(x) + f(z) + f(x+z) = 0 \text{ for all } z \in \mathbb{F}_{2^n}.$$

The following lemma establishes the relation between the trace transform and the dimension of the radical of a quadratic form.

Lemma 1. *(Helleseth and Kumar [6]) Let $f(x)$ be a quadratic Boolean function on $\mathbb{F}_{2^n}$. If the rank of $f(x)$ is $2h$, $2 \leq 2h \leq n$, then the distribution of the trace transform values is given by*

$$F(\lambda) = \begin{cases} 2^{n-h}, & 2^{2h-1} + 2^{h-1} times \\ 0, & 2^n - 2^{2h} times \\ -2^{n-h}, & 2^{2h-1} - 2^{h-1} times \end{cases}$$

where rank is the co-dimension of the radical of $f(x)$.

All the sequence families considered in this paper are constructed by using the trace function $a(x) = tr_1^n(x)$ and some quadratic form $b(x)$ as follows:

$$C = \{f_i(x) | 0 \leq i \leq 2^n, x \in \mathbb{F}_{2^n}^*\}$$

where

$$f_i(x) = \begin{cases} a(v_i x) + b(x), & 0 \leq i \leq 2^n - 1 \\ a(x), & i = 2^n. \end{cases}$$

and $\{v_0, v_1, ..., v_{2^n-1}\}$ is an enumeration of the elements in $\mathbb{F}_{2^n}$.

The correlation function between two sequences defined by $f_i(x)$ and $f_j(x)$ can be given by the function from $\mathbb{F}_{2^n}$ to the set of integers $\mathbb{Z}$ as

$$R_{i,j}(\delta) = \sum_{x \in \mathbb{F}_{2^n}^*} (-1)^{f_i(x) + f_j(\delta x)}$$

where $\delta \in \mathbb{F}_{2^n}^* = \mathbb{F}_{2^n} \setminus \{0\}$. $R_{i,j}(\delta)$ can be expressed as a trace transform

$$\begin{aligned} R_{i,j}(\delta) &= \sum_{x \in \mathbb{F}_{2^n}^*} (-1)^{tr_1^n([v_i + v_j]x) + g(x)} \\ &= -1 + \sum_{x \in \mathbb{F}_{2^n}} (-1)^{tr_1^n(x\lambda) + g(x)} \\ &= -1 + G(\lambda) \end{aligned}$$

where $g(x) = b(\delta x) + b(x)$ and $\lambda = v_i + v_j \in \mathbb{F}_{2^n}$.

Definition 2. *Let $\frac{n}{e} = m$ be odd. We define the boolean quadratic functions $p(x)$ and $q(x)$ by $p(x) = \sum_{l=1}^{\frac{n}{2}-1} tr_1^n(x^{2^l+1})$, $q(x) = \sum_{l=1}^{\frac{m}{2}-1} tr_1^n(x^{2^{el}+1})$.*

Lemma 2. *([4]) The associated symplectic form of $p(x)$ is*

$$B(x,z) = p(x) + p(z) + p(x+z) = tr_1^n[z(tr_1^n(x) + x)]. \quad (1)$$

Definition 3. *(Boztas and Kumar [4]) For an odd integer $n = 2k+1 \geq 3$, Boztas and Kumar introduced the following family G of Gold-like sequences*

$$g_i(x) = \begin{cases} tr_1^n(v_i x) + p(x), & 0 \leq i \leq 2^n - 1 \\ tr_1^n(x), & i = 2^n. \end{cases}$$

Theorem 1. *(Boztas and Kumar [4]) For the family G , the distribution of correlation values is given as follows:*

$$R_{i,j}(\delta) = \begin{cases} 2^n - 1, & 2^n + 1 \ times \\ -1, & 2^{3n-1} + 2^{2n} - 2^n - 2 \ times \\ -1 + 2^{k+1}, & 2^{2n-2}(2^{2k-1} + 2^{k-1}) \ times \\ -1 - 2^{k+1}, & 2^{2n-2}(2^{2k-1} - 2^{k-1}) \ times. \end{cases}$$

Lemma 3. *([7]) The associated symplectic form of $q(x)$ is*

$$B(x,z) = q(x) + q(z) + q(x+z) = tr_1^n[z(tr_e^n(x) + x)]. \quad (2)$$

Definition 4. *(Kim and No[7]) Let $\frac{n}{e} = m$ be an odd integer, where $m \geq 3$. Kim and No introduced the following sequences S which generalized the previous family*

$$s_i(x) = \begin{cases} tr_1^n(v_i x) + q(x), & 0 \leq i \leq 2^n - 1 \\ tr_1^n(x), & i = 2^n. \end{cases}$$

Theorem 2. *(Kim and No [7]) For the family S , the distribution of correlation values is given as follows:*

$$R_{i,j}(\delta) = \begin{cases} 2^n - 1, & 2^n + 1 \ times \\ -1, & (2^n - 2^{n-e}) + 1)(2^{2n} - 2) \ times \\ -1 + 2^{\frac{n+e}{2}}, & (2^{n-e-1} + 2^{\frac{n-e-2}{2}})(2^{2n} - 2) \ times \\ -1 - 2^{\frac{n+e}{2}}, & (2^{n-e-1} - 2^{\frac{n-e-2}{2}})(2^{2n} - 2) \end{cases}$$

In their calculation to find the rank the symplectic forms (1) and (2) have been used respectively. We have used those two symplectic form in rather modified form to construct a family based on two quadratic forms $p(\lambda x)$ and $q(\zeta x)$ [11].

Definition 5. *Let $\frac{n}{e} = m \geq 3$ be odd. We define the family $\mathcal{U}$ of binary sequences by*

$$u_i(x) = \begin{cases} tr_1^n(v_i x) + p(\lambda x) + q(\zeta x), & 0 \le i \le 2^n - 1 \\ tr_1^n(x), & i = 2^n. \end{cases}$$

where e is also odd, $\lambda, \zeta \in \mathbb{F}_{2^e}$ and $\lambda \neq 0, \lambda \neq \zeta$.

For the correlation property of the family $\mathcal{U}$, we have the following result.

Theorem 3. *([11])The family $\mathcal{U}$ has the following properties:*

1. *The maximal absolute value of the nontrivial correlation of family $\mathcal{U}$ is bounded by $R_{max} \le 1 + 2^{\frac{n+1}{2}}$ and so the family is optimal with respect to Sidelnikov bound.*
2. *The correlation distribution is as follows:*

$$R_{i,j}(\delta) = \begin{cases} 2^n - 1, & 2^n + 1 \text{ times} \\ -1, & 2^{3n-1} + 2^{2n} - 2^n - 2 \text{ times} \\ -1 + 2^{\frac{n+1}{2}}, & (2^{2n} - 2)(2^{n-2} + 2^{\frac{n-3}{2}}) \text{ times} \\ -1 - 2^{\frac{n+1}{2}}, & (2^{2n} - 2)(2^{n-2} - 2^{\frac{n-3}{2}}) \text{ times.} \end{cases}$$

Definition 6. *Let $\frac{n}{e} = m$ be even. We define the Boolean functions $p(x)$ and $q(x)$ by $p(x) = \sum_{l=1}^{\frac{n}{2}-1} tr_1^n(x^{2^l+1})$, $q(x) = \sum_{l=1}^{\frac{m}{2}-1} tr_1^n(x^{2^{el}+1})$.*

Definition 7. *(Udaya [13]) For an even integer $n = 2k \ge 4$, Udaya introduced the following family G*

$$g_i(x) = \begin{cases} tr_1^n(v_i x) + p(x) + tr_1^{\frac{n}{2}}(x^{2^{\frac{n}{2}}+1}), & 0 \le i \le 2^n - 1 \\ tr_1^n(x), & i = 2^n. \end{cases}$$

Theorem 4. *(Udaya [13]) For the family G , the distribution of correlation values is given as follows:*

$$R_{i,j}(\delta) = \begin{cases} 2^n - 1, & 2^n + 1 \text{ times} \\ -1, & 2^{2n-1}(2^{n-1} + 2^{n-2}) + 2^{2n} - 2 \text{ times} \\ -1 + 2^k, & (2^{2n-1} - 2)(2^{n-1} + 2^{k-1}) \text{ times} \\ -1 - 2^k, & (2^{2n-1} - 2)(2^{n-1} - 2^{k-1}) \text{ times} \\ -1 + 2^{k+1}, & 2^{2n-1}(2^{n-3} + 2^{k-2}) \text{ times} \\ -1 - 2^{k+1}, & 2^{2n-1}(2^{n-3} - 2^{k-2}) \text{ times.} \end{cases}$$

Definition 8. *(Kim and No [7]) Let $\frac{n}{e} = m$ be an even integer, where $m \ge 4$. Kim and No introduced the following sequences S with six-valued correlations.*

$$s_i(x) = \begin{cases} tr_1^n(v_i x) + q(x) + tr_1^{\frac{n}{2}}(x^{2^{\frac{n}{2}}+1}), & 0 \le i \le 2^n - 1 \\ tr_1^n(x), & i = 2^n. \end{cases}$$

Theorem 5. *([7]) For the family S , the distribution of correlation values is given as follows:*

$$R_{i,j}(\delta) = \begin{cases} 2^n - 1, & 2^n + 1 \text{ times} \\ -1, & 2^{2n-e}(2^n - 2^{n-2e}) + (2^{2n} - 2) \text{ times} \\ -1 + 2^{\frac{n+2e}{2}}, & 2^{2n-e}(2^{n-2e-1} + 2^{\frac{n-2e-2}{2}}) \text{ times} \\ -1 - 2^{\frac{n+2e}{2}}, & 2^{2n-e}(2^{n-2e-1} - 2^{\frac{n-2e-2}{2}}) \\ -1 + 2^{\frac{n}{2}}, & (2^{2n} - 2^{2n-e} - 2)(2^{n-1} + 2^{\frac{n}{2}-1}) \text{times} \\ -1 - 2^{\frac{n}{2}}, & (2^{2n} - 2^{2n-e} - 2)(2^{n-1} - 2^{\frac{n}{2}-1}) \text{times.} \end{cases}$$

In this paper we introduce a new family U which is a combination of G and S.

Definition 9. *Let $\frac{n}{e} = m \ge 4$ be even. We define the family U of binary sequences by*

$$u_i(x) = \begin{cases} tr_1^n(v_i x) + p(x) + q(x), & 0 \le i \le 2^n - 1 \\ tr_1^n(x), & i = 2^n. \end{cases}$$

For the correlation property of the family U, we have the following result.

Theorem 6. *([12]) The distribution of correlation values of the family U is given as when e is odd*

Correlation $(R_{i,j}(\delta))$	*Number of times it appears*
$2^n - 1$	$2^n + 1$
-1	$2^{3n} + 2^{2n} - 2^{n+1} + 2^e + 2^{n+2e-1}(2^{e-1} - 2^{n-1} - 1) - 2$
$-1 + 2^{n-\frac{e-1}{2}}$	$(2^{e-2} + 2^{\frac{e-3}{2}})(2^{n+e} - 2)$
$-1 - 2^{n-\frac{e-1}{2}}$	$(2^{e-2} - 2^{\frac{e-3}{2}})(2^{n+e} - 2)$
$-1 + 2^{n-e+1}$	$(2^{2e-3} + 2^{e-2})(2^{2n} - 2^{n+e})$
$-1 - 2^{n-e+1}$	$(2^{2e-3} - 2^{e-2})(2^{2n} - 2^{n+e})$

and when e is even

Correlation $(R_{i,j}(\delta))$	*Number of times it appears*
$2^n - 1$	$2^n + 1$
-1	$2^{3n} + 2^{2n} - 2^{n+1} + 2^{e+1} + 2^{n+e-2}(3 \cdot 2^{e-1} - 2^{n+2} - 3) - 2$
$-1 + 2^{n-\frac{e}{2}}$	$(2^{e-1} + 2^{\frac{e-2}{2}})(2^{n+e-1} + 2^n - 2)$
$-1 - 2^{n-\frac{e}{2}}$	$(2^{e-1} - 2^{\frac{e-2}{2}})(2^{n+e-1} + 2^n - 2)$
$-1 + 2^{n-\frac{e-2}{2}}$	$(2^{e-3} + 2^{\frac{e-4}{2}})(2^{n+e-1} - 2^n)$
$-1 - 2^{n-\frac{e-2}{2}}$	$(2^{e-3} - 2^{\frac{e-4}{2}})(2^{n+e-1} - 2^n)$
$-1 + 2^{n-e}$	$(2^{2e-1} + 2^{e-1})(2^{2n} - 2^{n+e})$
$-1 - 2^{n-e}$	$(2^{2e-1} - 2^{e-1})(2^{2n} - 2^{n+e})$

Definition 10. *Let n be odd, $\delta_1 \in \mathbb{F}_{2^n}\backslash\{0,1\}$. We define* $p(x) = \sum_{l=1}^{\frac{n-1}{2}} tr_1^n(x^{2^l+1} + (\delta_1 x)^{2^l+1})$,
$q(x) = p(x) + p(\delta_2 x)$, *where* $\delta_2 \in \mathbb{F}_{2^n}\backslash\{0,1\}$ *and* $\delta_1 \neq \delta_2$.

Using the rank of $p(x)$ and $q(x)$, Wang and Qi [14] has introduced the following result.

Theorem 7. *([14] Let n be odd, $\delta_1 \in \mathbb{F}_{2^n}\backslash\{0,1\}$, $p(x) = \sum_{l=1}^{\frac{n-1}{2}} tr_1^n(x^{2^l+1} + (\delta_1 x)^{2^l+1})$. Then the family $S_1 = \{s_{1,j} | j = 0,1,\ldots,2^n\}$ given by*

$$s_{1,j}(x) = \begin{cases} tr_1^n(v_j x) + p(x), & 0 \leq i \leq 2^n - 1 \\ tr_1^n(x), & i = 2^n. \end{cases}$$

has the following correlation distribution:

Correlation $(R_{i,j}(\delta))$	*Number of times it appears*
$2^n - 1$	$2^n + 1$
-1	$2^{3n-1} + 2^{3n-4} - 2^{3n-6} + 2^{2n} - 2^n - 2$
$-1 + 2^{\frac{n+1}{2}}$	$(2^{n-2} + 2^{\frac{n-3}{2}})(2^{2n} - 2^{2n-3} - 2)$
$-1 - 2^{\frac{n+1}{2}}$	$(2^{n-2} - 2^{\frac{n-3}{2}})(2^{2n} - 2^{2n-3} - 2)$
$-1 + 2^{\frac{n+3}{2}}$	$(2^{n-4} + 2^{\frac{n-5}{2}})2^{2n-3}$
$-1 - 2^{\frac{n+3}{2}}$	$(2^{n-4} - 2^{\frac{n-5}{2}})2^{2n-3}$

Definition 11. *Let n be even, $\delta_1 \in \mathbb{F}_{2^n}\backslash\{0,1\}$. We define* $p_1(x) = tr_1^n(x^{2^{n/2}+1} + (\delta_1 x)^{2^{n/2}+1}) + \sum_{l=1}^{\frac{n}{2}-1} tr_1^n(x^{2^l+1} + (\delta_1 x)^{2^l+1})$,
$q_1(x) = p_1(x) + p(\delta_2 x)$, *where* $\delta_2 \in \mathbb{F}_{2^n}\backslash\{0,1\}$ *and* $\delta_1 \neq \delta_2$.

Using the rank of $p_1(x)$ and $q_1(x)$, Wang and Qi [14] has introduced another new family.

Theorem 8. *Let n be even, $\delta_1 \in \mathbb{F}_{2^n}\backslash\{0,1\}$, $q(x) = tr_1^n(x^{2^{n/2}+1} + (\delta_1 x)^{2^{n/2}+1}) + \sum_{l=1}^{\frac{n}{2}-1} tr_1^n(x^{2^l+1} + (\delta_1 x)^{2^l+1})$, Then the family $S_2 = \{s_{2,j} | j = 0,1,\ldots,2^n\}$ given by*

$$s_{2,j}(x) = \begin{cases} tr_1^n(v_j x) + p(x), & 0 \leq i \leq 2^n - 1 \\ tr_1^n(x), & i = 2^n. \end{cases}$$

has the following correlation distribution:

Correlation $(R_{i,j}(\delta))$	*Number of times it appears*
$2^n - 1$	$2^n + 1$
-1	$2^{3n-1} - 2^{3n-3} + 2^{3n-6} - 2^{3n-8} + 2^{2n} - 2$
$-1 + 2^{\frac{n}{2}}$	$(2^{n-1} + 2^{\frac{n}{2}-1})(2^{2n-1} - 2)$
$-1 - 2^{\frac{n}{2}}$	$(2^{n-1} - 2^{\frac{n}{2}-1})(2^{2n-1} - 2)$
$-1 + 2^{\frac{n}{2}+1}$	$(2^{n-3} + 2^{\frac{n}{2}-2})(2^{2n-1} - 2^{2n-4})$
$-1 - 2^{\frac{n}{2}+1}$	$(2^{n-3} - 2^{\frac{n}{2}-2})(2^{2n-1} - 2^{2n-4})$
$-1 + 2^{\frac{n}{2}+2}$	$(2^{n-5} + 2^{\frac{n}{2}-3})2^{2n-4}$
$-1 - 2^{\frac{n}{2}+2}$	$(2^{n-5} - 2^{\frac{n}{2}-3})2^{2n-4}$

if $tr_1^n((1+\delta_1)^{-1}) = 0$ *and*

Correlation $(R_{i,j}(\delta))$	*Number of times it appears*
$2^n - 1$	$2^n + 1$
-1	$2^{3n-1} - 2^{3n-3} + 2^{3n-6} - 2^{3n-8} + 2^{2n} - 2$
$-1 + 2^{\frac{n}{2}}$	$(2^{n-1} + 2^{\frac{n}{2}-1})2^{2n-1}$
$-1 - 2^{\frac{n}{2}}$	$(2^{n-1} - 2^{\frac{n}{2}-1})2^{2n-1}$
$-1 + 2^{\frac{n}{2}+1}$	$(2^{n-3} + 2^{\frac{n}{2}-2})(2^{2n-1} - 2)$
$-1 - 2^{\frac{n}{2}+1}$	$(2^{n-3} - 2^{\frac{n}{2}-2})(2^{2n-1} - 2)$

if $tr_1^n((1+\delta_1)^{-1}) = 1$.

III. CONCLUSION

In this article we have seen a quick survey on families of binary m-sequences with 3,4 and more non-trivial correlation values. More results can be found in recent literature. But in order to achieve bigger linear span many families of quartenary sequences have been introduced.

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